Automatic Control

Chapter three

Signal flow graph

By

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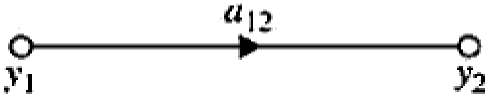
Block diagrams and SFG

Signal Flow Graph (SFG)

A signal-flow graph (SFG) may be regarded as a simplified version of a block diagram

Basic Elements of an SFG

Node: represent variables
Arrow head: the direction of the signal flow
Line: where the signal flow
Transfer function (a): the relation between the variables
Variables (y)



Mathematical relation

$$y_2 = a_{12} y_1$$

Block diagrams and SFG

Signal Flow Graph (SFG)

Example 3-2-1 Golnaraghi (2010

Draw the SFG for the following system of linear algebraic equations

$$y_2 = a_{12}y_1 + a_{32}y_3$$

$$y_3 = a_{23}y_2 + a_{43}y_4$$

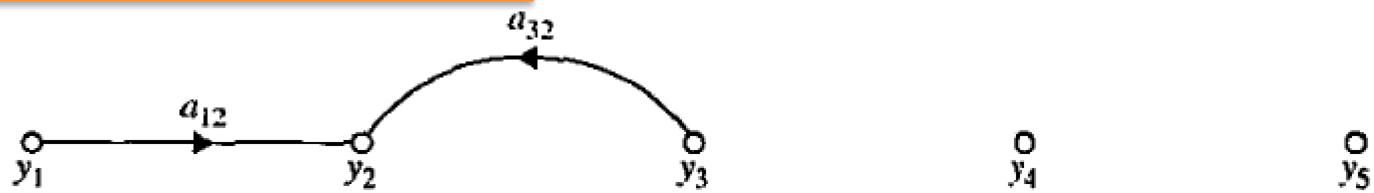
$$y_4 = a_{24}y_2 + a_{34}y_3 + a_{44}y_4$$

$$y_5 = a_{25}y_2 + a_{45}y_4$$

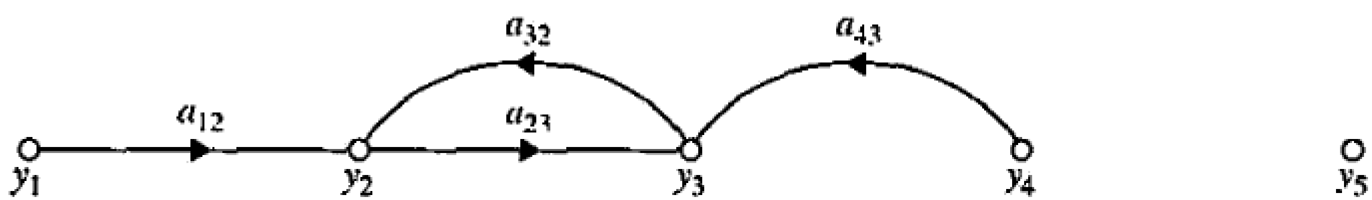
Block diagrams and SFG

Example 3-2-1 Golnaraghi (2010

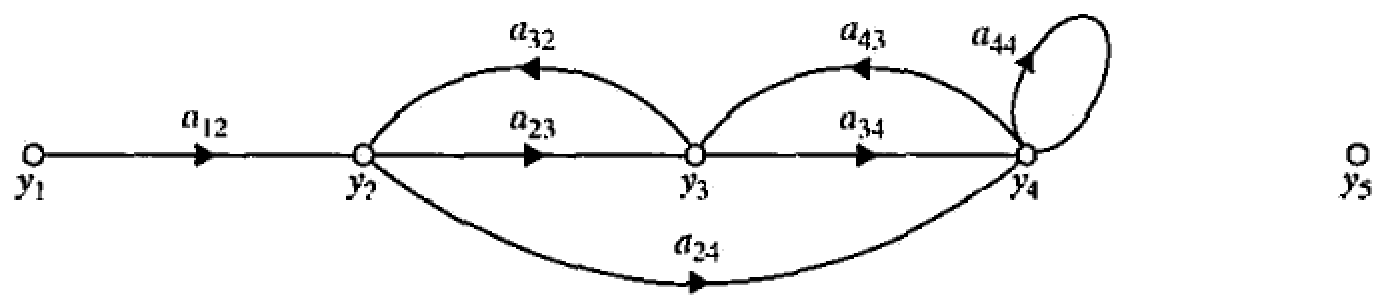
Solution



(a) $y_2 = a_{12}y_1 + a_{32}y_3$



(b) $y_2 = a_{12}y_1 + a_{32}y_3 \quad y_3 = a_{23}y_2 + a_{43}y_4$



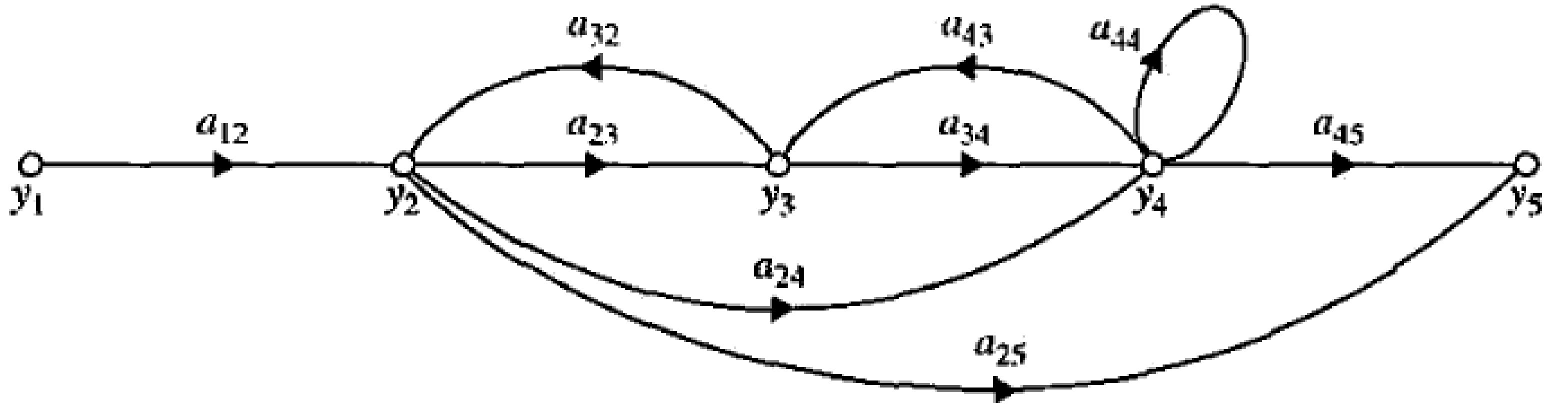
(c) $y_2 = a_{12}y_1 + a_{32}y_3 \quad y_3 = a_{23}y_2 + a_{43}y_4 \quad y_4 = a_{24}y_2 + a_{34}y_3 + a_{44}y_4$

A u t o m a t i c C o n t r o l

Block diagrams and SFG

Example 3-2-1 Golnaraghi (2010

Solution



(d) Complete signal-flow graph

Block diagrams and SFG

Basic Properties of SFG

- *SFG applies only to linear systems.*
- *The equations for which an SFG is drawn must be algebraic equations in the form of cause-and-effect.*
- *Nodes are used to represent variables. Normally, the nodes are arranged from left to right, from the input to the output, following a succession of cause-and-effect relations through the system.*
- *Signals travel along branches only in the direction described by the arrows of the branches.*
- *The branch directing from node y_k to y_j represents the dependence of y_k upon y_j but not the reverse.*
- *A signal y_k traveling along a branch between y_k to y_j is multiplied by the gain of the branch a_{kj}, so a signal $a_{kj}y_k$ is delivered at y_j.*

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Block diagrams and SFG

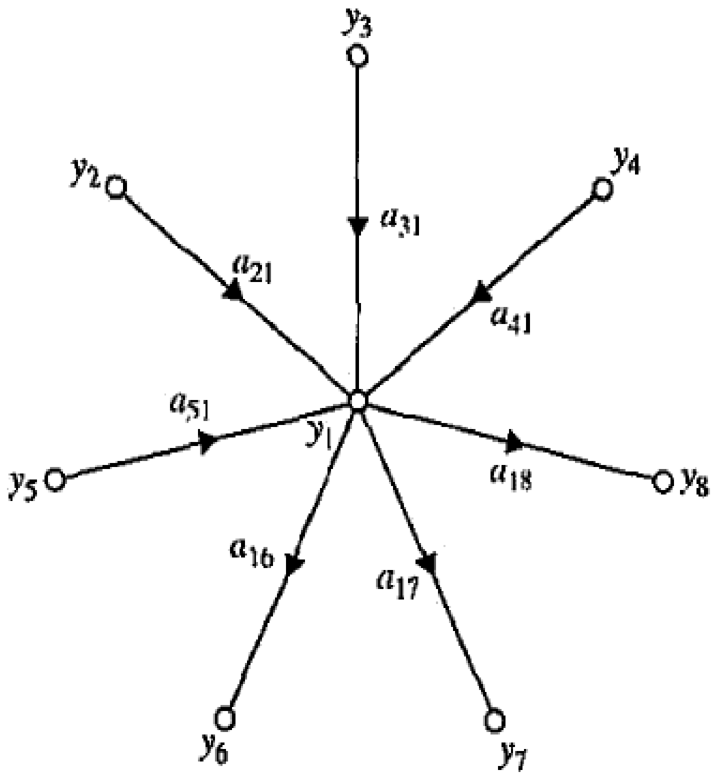
Definitions of SFG Terms

- *Input Node (Source): An input node is a node that has only outgoing branches*
- *Output Node (Sink): An output node is a node that has only incoming branches*
- *Path: A path is any collection of a continuous succession of branches traversed in the same direction.*
- *Forward Path: A forward path is a path that starts at an input node and ends at an output node and along which no node is traversed more than once*
- *Path Gain: The product of the branch gains encountered in traversing a path is called the path gain*
- *Loop: A loop is a path that originates and terminates on the same node and along which no other node is encountered more than once.*
- *Forward-Path Gain: The forward-path gain is the path gain of a forward path.*
- *Loop Gain: The loop gain is the path gain of a loop.*
- *Nontouching Loops: Two parts of an SFG are nontouching if they do not share a common node.*

Block diagrams and SFG

SFG Algebra

- The value of the variable represented by a node is equal to the sum of all the signals entering the node.
- The value of the variable represented by a node is transmitted through all branches leaving the node



$$y_1 = a_{21}y_2 + a_{31}y_3 + a_{41}y_4 + a_{51}y_5$$

$$y_6 = a_{16}y_1$$
$$y_7 = a_{17}y_1$$
$$y_8 = a_{18}y_1$$

Block diagrams and SFG

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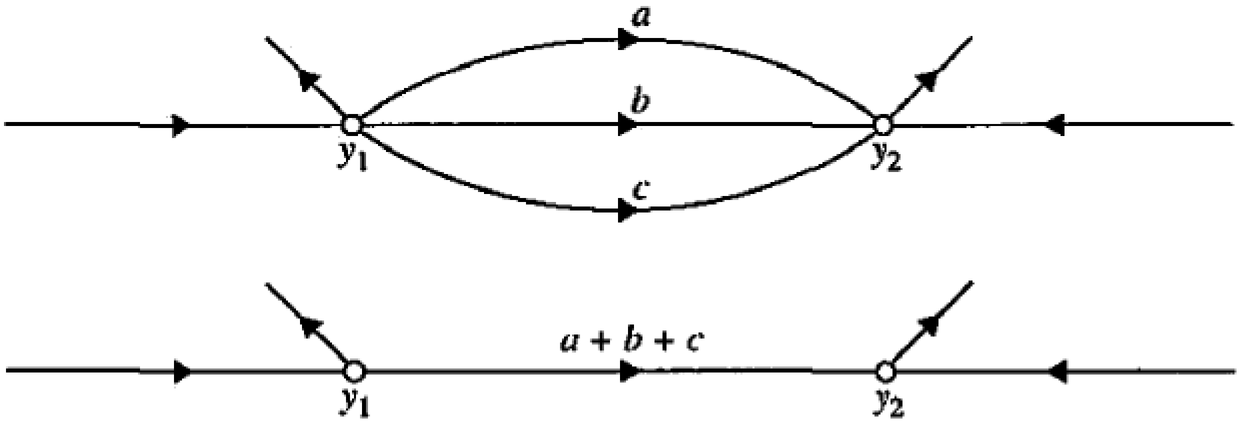
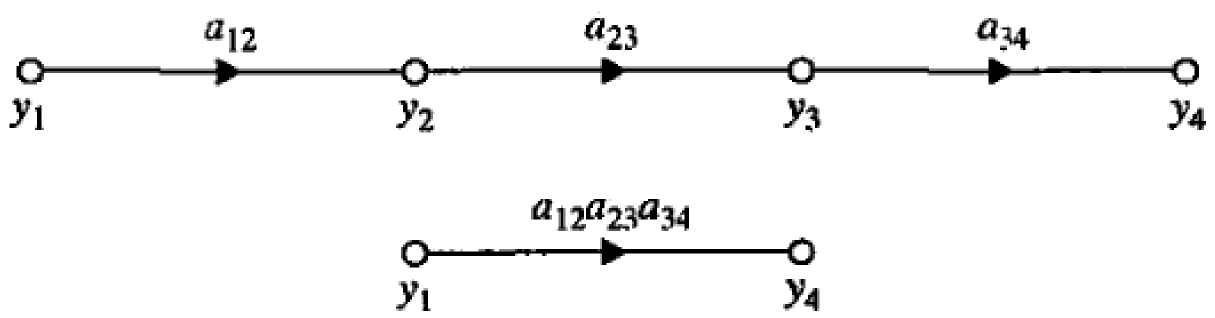
SFG Algebra

- Parallel branches in the same direction connecting two nodes can be replaced by a single branch with gain equal to the sum of the gains of the parallel branches



- A series connection of unidirectional branches can be replaced by a single branch with gain equal to the product of the branch gains.



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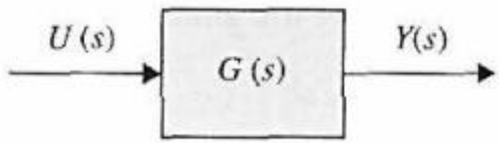
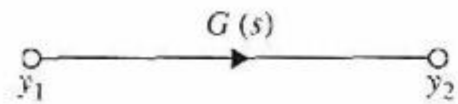
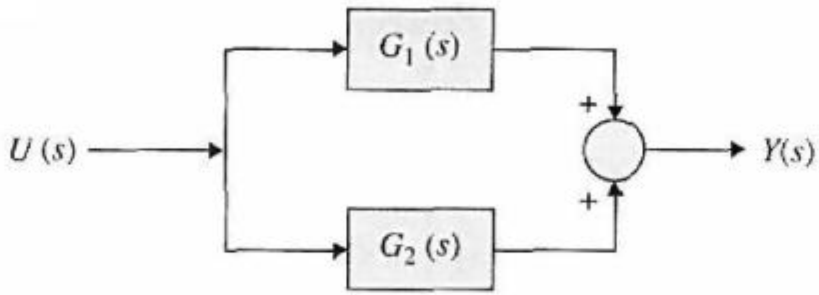
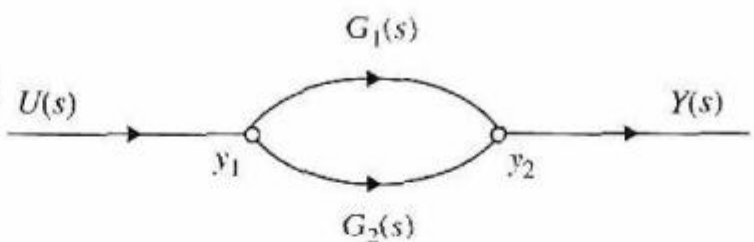
Block diagrams and SFG

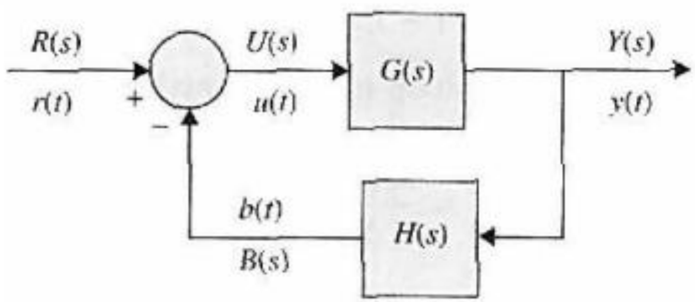
Relation between SFG and block diagram

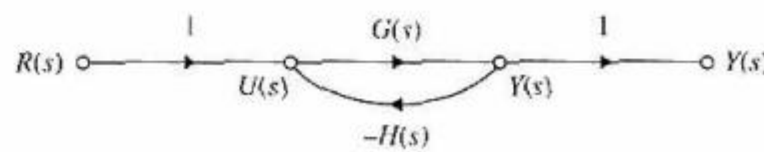








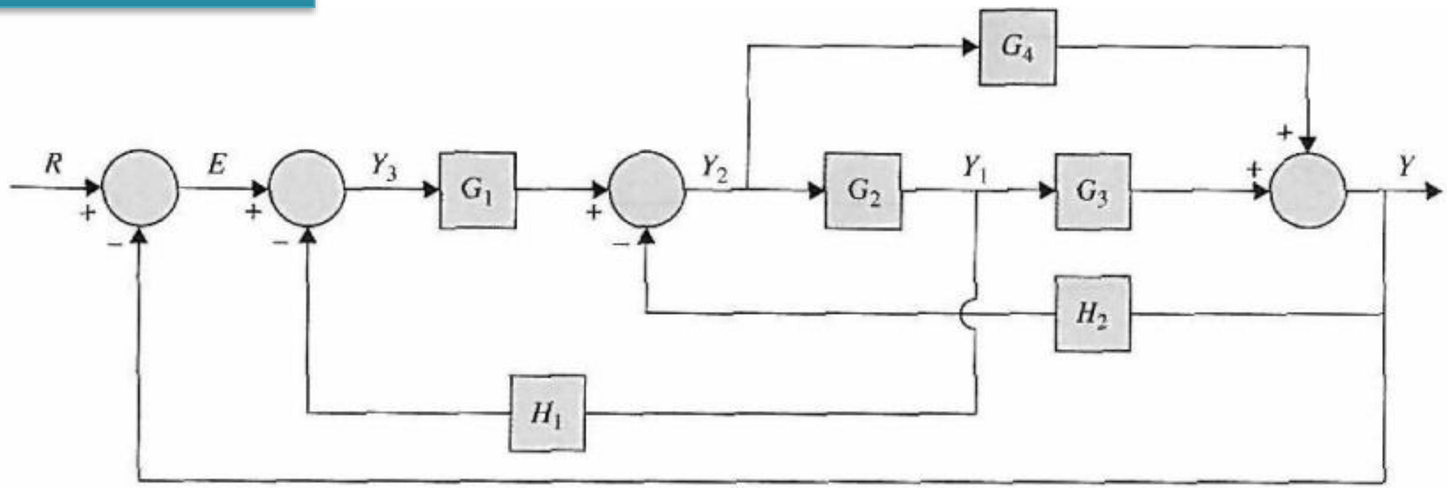




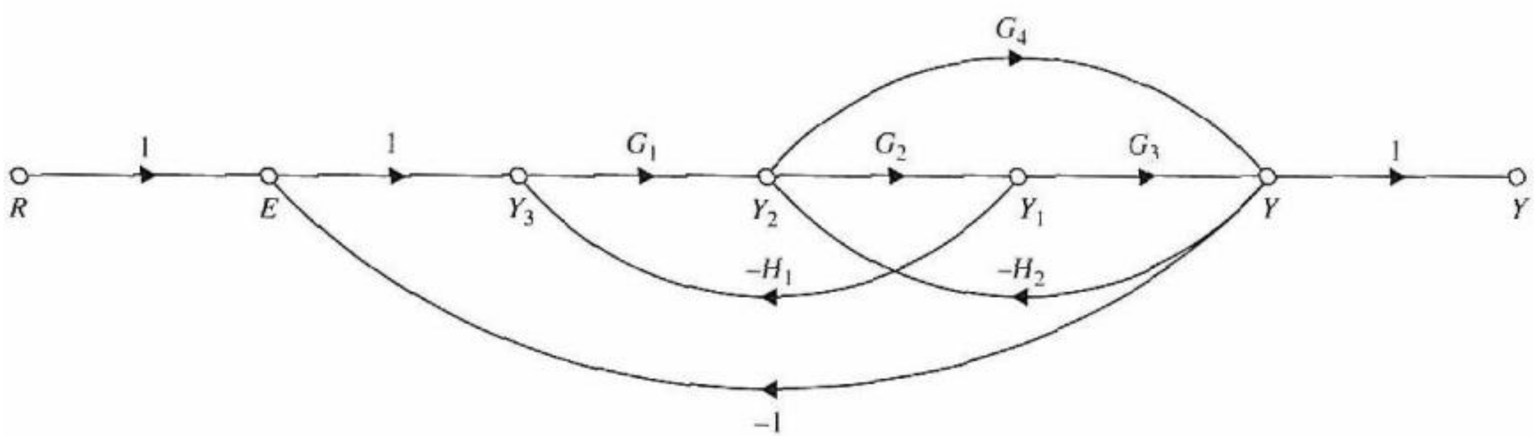
Block diagrams and SFG

Relation between SFG and block diagram

Example



(a)



Block diagrams and SFG

Gain Formula for SFG

To find the relation between the SFG input and output, we can use the gain formula for SFG:

$$M = \frac{y_{out}}{y_{in}} = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta}$$

where

y_{in}= input-node variable

y_{out} = output-node variable

M = gain between yin and y_{out}

N = total number of forward paths between y_{in} and y_{out}

M_k = gain of the kth forward paths between y_{in} and y_{out}

$$\Delta = 1 - \sum_i L_{i1} + \sum_j L_{j2} - \sum_k L_{k3} + \ldots$$

Δ= 1 - (sum of the gains of all individual loops) + (sum of products of gains of all possible combinations of two nontouching loops) — (sum of products of gains of all possible combinations of three nontouching loops) +...

Δk is the Δ for that part of the SFG that is nontouching with the kth forward path.

Block diagrams and SFG

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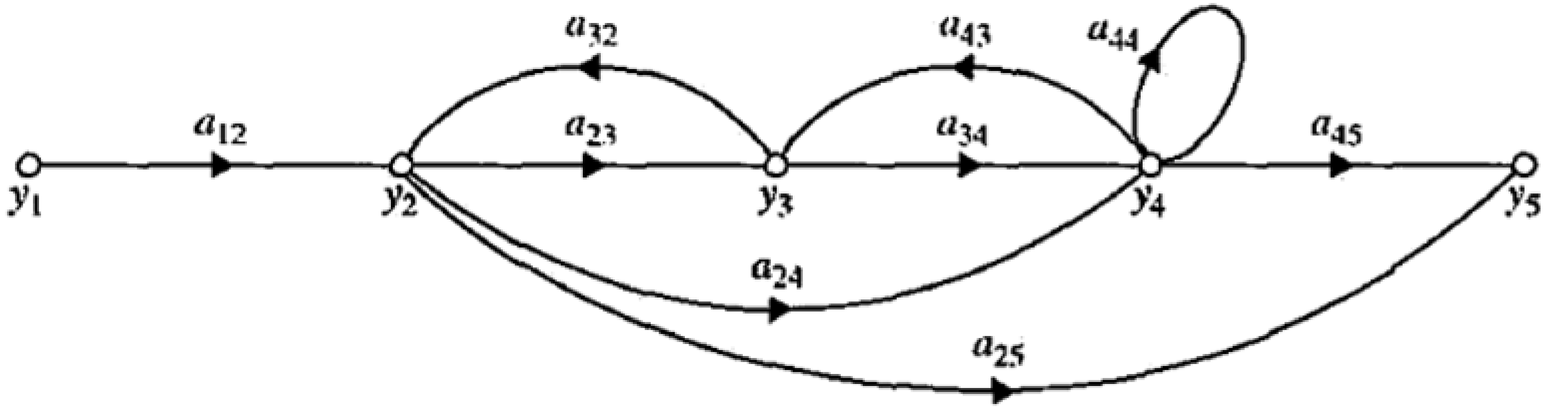
Block diagrams and SFG

Example 3-2-3 Golnaraghi (2010

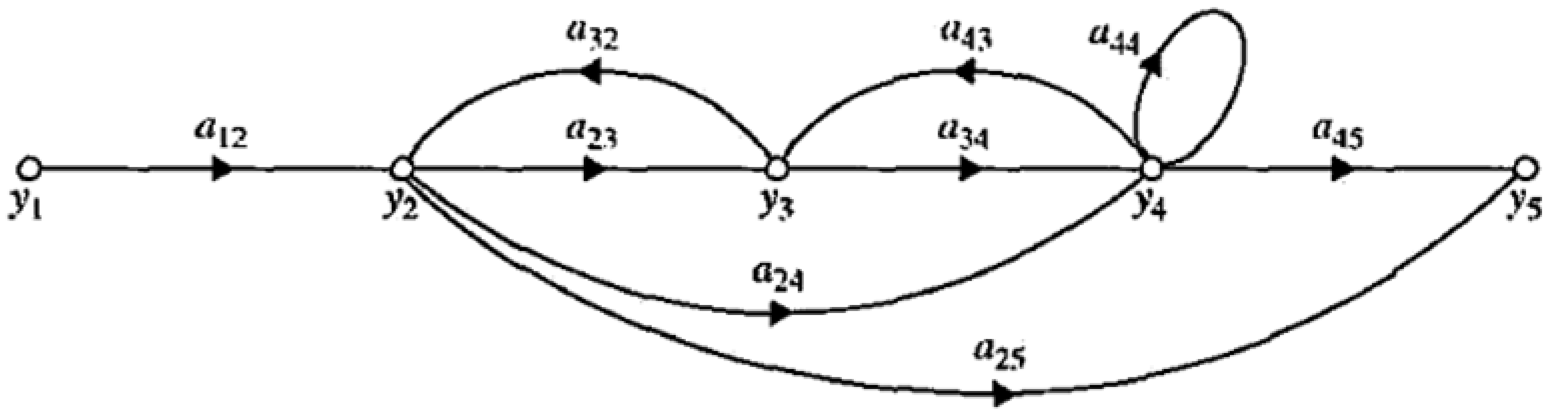
determine the gain between y_1 and y_5 using the gain formula for the following SFG.



Block diagrams and SFG

Example 3-2-3 Golnaraghi (2010

Solution



The three forward paths between y_1 and y_5 *and the forward-path gains are*

$$M_1 = a_{12}a_{23}a_{34}a_{45} \quad \text{Forward path:} \quad y_1 - y_2 - y_3 - y_4 - y_5$$
$$M_2 = a_{12}a_{25} \quad \text{Forward path:} \quad y_1 - y_2 - y_5$$
$$M_3 = a_{12}a_{24}a_{45} \quad \text{Forward path:} \quad y_1 - y_2 - y_4 - y_5$$

Block diagrams and SFG

Example 3-2-3 Golnaraghi (2010

Solution

There is only one pair of nontouching loops; that is, the two loops are:

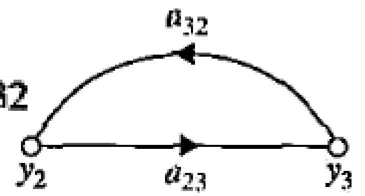
$$y_2 - y_3 - y_2 \text{ and } y_4 - y_4.$$

Thus, the product of the gains of the two nontouching loops is

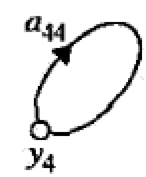
$$L_{12} = a_{23}a_{32}a_{44}.$$

The four loops of the SFG

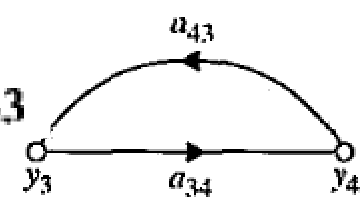
$$L_{11} = a_{23}a_{32}$$



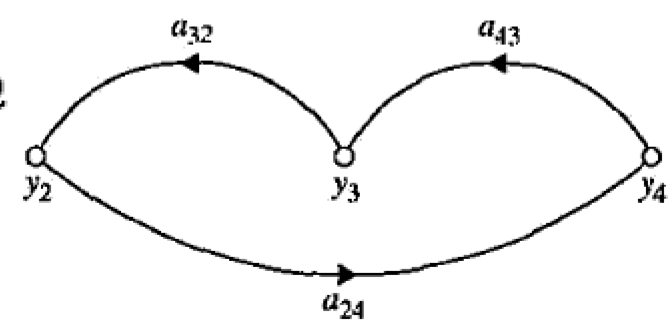
$$L_{41} = a_{44}$$



$$L_{21} = a_{34}a_{43}$$



$$L_{31} = a_{24}a_{43}a_{32}$$



Block diagrams and SFG

Example 3-2-3 Golnaraghi (2010

Solution

All the loops are in touch with forward paths M_1 *and* M_3*. Thus,* $\Delta_1 = \Delta_3 = 1$*. Two of the loops are not* in touch with forward path M_2*. These loops are* $y_3 — y_4 – y_3$ *and* $y_4 — y_4$*. Thus*

$$\Delta_2 = 1 - a_{34}a_{43} - a_{44}.$$

Substitute in gain formula

$$M = \frac{y_5}{y_1} = \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3}{\Delta}$$

$$M = \frac{(a_{12}a_{23}a_{34}a_{45}) + (a_{12}a_{25})(1 - a_{34}a_{43} - a_{44}) + a_{12}a_{24}a_{45}}{1 - (a_{23}a_{32} + a_{34}a_{43} + a_{24}a_{32}a_{43} + a_{44}) + a_{23}a_{32}a_{44}}$$